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ORIGINAL PAPER

Multiple Model Adaptive Attitude Control of LEO Satellite with Angular Velocity Constraints

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Abstract
In this paper, the multiple model adaptive control is utilized to improve the transient response of attitude control system for a rigid spacecraft. An adaptive output feedback control law is proposed for attitude control under angular velocity constraints and its almost global asymptotic stability is proved. The multiple model adaptive control approach is employed to counteract large uncertainty in parameter space of the inertia matrix. The nonlinear dynamics of a low earth orbit satellite is simulated and the proposed control algorithm is implemented. The reported results show the effectiveness of the suggested scheme.

Keywords Adaptive control · Satellite attitude control · Multiple model

1 Introduction

Satellites have such a significant role in today's life that we almost cannot imagine our everyday life without satellites services. The mission of a satellite and a space project, in general, relies on its payload and the performance of most of common payloads in space projects is tightly related to the performance of the attitude control system (ACS). The ACS is responsible for reorienting the spacecraft to achieve desired orientation or attitude and counteract various disturbances present in the space environment.

Taking into account some demanding applications such as stereo imaging the need for ACSs of a higher performance is evident. In this regard, this work focuses on improving the transient response of ACS of a LEO satellite subjected to uncertainty in mass properties. This kind of uncertainty is usual in satellites because of various facts including fuel consumption and deployable or sun-tracking solar arrays.

The attitude control problem is also of great interest in areas other than space projects such as aerial vehicles, robotics systems and submarine vehicles. There have been vast amounts of research dealing with various aspects of this problem during past few decades. An example of a comprehensive reference on spacecraft attitude control could be Ref. [1], while an analytical treatment to the subject can be found in Ref. [2], and Ref. [3] is an application oriented book. An in-depth treatment of attitude determination is given in Ref. [4]. This reference covers fundamental concepts and mathematical basis for spacecraft attitude determination and control and develops them to practical algorithms. A detailed list of references through 1991 is given by Wen [5]. In this reference, a feature inherent in quaternion for describing the configuration space of rigid body attitude motion that is double covering of the attitude space was pointed out. The consequent problems of this feature are the so-called unwinding phenomenon, the impossibility of globally stabilizing the attitude using a continuous controller which led to the introduction of "*almost*" global stability notion, the need for a path lifting mechanism and using sort of memory in control law. To be more specific, the state space of attitude motion, SO(3), which is the set of all orthogonal 3×3 matrices with unit determinant, is a boundaryless compact manifold and is not a vector space. On the other hand quaternion representation of attitude is the set of all vectors in $\Re^4$ with unit magnitude and it double covers the set SO(3). These problems are now well understood and reported in Refs. [6,7] among others.

The problem of stabilizing spacecraft attitude has been considered for a long time by many researchers and there are a variety of proposed techniques such as Refs. [8,9] to cite main works. In these works, a PD-like controller with a linear structure is used. The proportional term includes a measure of attitude error [5], while the derivative term uses angular velocity for damping purposes. Various output feedback

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controllers have been also proposed; in contrast to Ref. [10] which uses dynamic observer to establish output feedback, Lizarralde proposed a passivity-based lead filter to generate pseudo-velocities to be used in control law [11]. This scheme which thereafter was also incorporated in many works such as Refs. [12,13], though this approach eliminates the direct use of angular velocity in control law; as there does not exist any device to measure the attitude directly, the need for angular velocity measurement might not be eliminated. A finite time state observer together with a finite time control law in terms of MRPs has been proposed in Ref. [14] that constitute a finite time output feedback attitude control scheme. This scheme is also applicable to a more general class of second-order nonlinear systems. The stability analysis for both observer and control law is also presented. Nonlinear control techniques including Refs. [15–17] have been used successfully in the context of attitude control. Robust control has been also exploited in Refs. [15,18] to treat uncertainty in inertia matrix, actuators misalignment, and different disturbances.

Adaptive attitude control seems to have great potential for satisfying spacecraft attitude control problem requirements. A wide class of adaptive control schemes has been proposed. In Ref. [19], an adaptive trajectory tracking controller is presented for a large class of nonlinear mechanical systems especially the rigid body attitude control problem. The response of closed loop system in this approach converges asymptotically to a specified linear PID response. An adaptive attitude controller subject to constraints on angular velocity is proposed in Ref. [20]. In this control law the constraints on angular velocity components are explicitly used in the controller formulation. In Ref. [21] a model reference adaptive controller is developed for spacecraft rendezvous and docking problem. In this work a passivity based lead filter similar to that of Ref. [12] is used to achieve output feedback control. In Ref. [22] an adaptive attitude controller with finite-time convergence which uses a nonsingular terminal sliding surface is developed. Two robust adaptive finite time control laws using back-stepping method are presented in Ref. [23] which are robust against external disturbances. In this work rotation matrix is incorporated for describing the attitude of the rigid spacecraft to avoid pitfalls associated with other coordinates which are singular and/or non-unique.

All of the above-mentioned adaptive controllers are based on certainty-equivalence principle; in turn, they consider a deterministic control law and combine it with an appropriate parameter adaptation law to achieve an adaptive control law. The resulting closed loop system is nonlinear time-varying and due to parameter adaptation apart from the actual values has inferior performance to the deterministic case. To compensate for this imperfection, authors in Ref. [24] recover the performance of deterministic closed loop system by including a stable attracting manifold in adaptation scheme. Another popular and more general solution to overcome the drawbacks of classical adaptive control which we adopted in this work is the multiple model and switching approach. In classical adaptive control, the plant is supposed to have unknown constant parameters and because of this in the case of abrupt change or large uncertainty in the parameters, classical adaptive control leads to a poor performance especially from a transient response point of view. As a solution to these problems, multiple model and switching approach attracted interests from the early ages of adaptive control. Implementation of this approach is presented by Maybeck [25] for the aircraft flight control problem. Multiple model adaptive control with switching and tuning with stability proof for special cases is introduced in Ref. [26]. Fekri [27] tries to give a methodology for designing multiple model adaptive controllers that guarantee a superior performance and stability properties in comparison with the best non-adaptive controllers. They have introduced the notion of robust multiple model adaptive control (RMMAC) where its stability is addressed by Hassani [28]. A weighted multiple model control for a discrete time linear plant with stability proof is proposed in Ref. [29] which guarantees the convergence of the weight of the closest model to the plant to 1 and the others to 0.

The proposed method in this paper is adopted from Ref. [30] which uses smaller number of models and provides an estimate of the plant parameter which depends on the collective outputs of all the models. Our focus in this paper is on attitude control under angular velocity constraints, since in spite of its practical applications there is much lower works on it than other problems in attitude control. Angular velocity constraints may have occurred in the cases such as low-rate gyros, in-flight refueling, and spacecraft docking. In Ref. [31], an integrator back-stepping technique for a dynamical system under angular velocity constraint is proposed and a Lyapunov function including a logarithmic term is introduced to deal with angular velocity bounds. Jianbo [32] introduced a nonlinear controller with actuator and slew rate saturation. A robust nonlinear control, again using the Lyapunov function including logarithmic term, was proposed by Hu [18]. The control law in this work uses feedback linearization to cancel gyroscopic term in attitude dynamics which is not new in attitude control and might not be desirable.

The contribution of this work is threefold. First, almost global asymptotic stability of a control law for attitude control under angular velocity constrains is rigorously proved. Second, the output feedback variant of this control law is presented and third, the transient response of the proposed control law is significantly improved exploiting multiple model approach.

This paper is organized as follows. In Sect. 2 the mathematical model of spacecraft attitude is stated. The proposed multiple model adaptive attitude control is presented in Sects.

3 and 4 describes the LEO satellite simulator used to evaluate the proposed controller and simulation results are presented therein. Finally, we conclude the paper in Sect. 5 and some future study issues are stated.

2 Mathematical Model of Spacecraft Attitude

In this section, the mathematical model of a rigid spacecraft is introduced. This model consists of spacecraft dynamics and its kinematics equation. Dynamics equation is described by the well-known Euler's moment equation and it concerns the act of torques on the rigid body rotational motion. Kinematics equation describes relationship between velocity and position-related quantities (attitude in rotational motion) regardless of torques acting on the body. While there are many representations for the attitude of a rigid body such as Euler angles, Rodrigues parameters, and modified Rodrigues parameters (MRPs); quaternions (also called Euler symmetric parameters) are most common since they are singularity-free and lead to a linear kinematics equation.

The quaternion vector representing the attitude of body frame with respect to inertial frame is introduced as

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_{1:3} \\ q_4 \end{bmatrix} \tag{1}$$

where $\mathbf{q}_{1:3}$ is the vector part and q_4 is the scalar part of the quaternion vector. Kinematics of a rigid body is given by the following equation [4]:

$$\dot{\mathbf{q}} = \frac{1}{2}\boldsymbol{\Omega}(\boldsymbol{\omega})\mathbf{q} \tag{2}$$

where $\boldsymbol{\omega}$ is the angular velocity of body frame with respect to inertial frame expressed in the body frame and

$$\boldsymbol{\Omega}(\boldsymbol{\omega}) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix}. \tag{3}$$

Euler's moment equation gives the nonlinear three-axis dynamics equation of a rigid body as

$$\mathbf{I}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega} \times \mathbf{I}\boldsymbol{\omega} + \boldsymbol{u} \tag{4}$$

where $\mathbf{I} \in \Re^{3\times3}$ is the symmetric positive definite inertia matrix of the rigid body, $\mathbf{u} \in \Re^3$ is the control input, and $\times$ denotes cross product operation.

3 Multiple Model Adaptive Attitude Controller

In this section, the proposed multiple model adaptive output feedback attitude control scheme is described. First, a modified variant of adaptive control law presented in Ref. [20], which explicitly takes into account angular velocity bounds, is introduced. Then the need for direct angular velocity measurement is eliminated using passivity-based lead filter, and utilizing the resulting control law as a core controller to improve its transient response using multiple model and switching approach to adaptive control.

Let $\mathbf{q}$ be the instantaneous attitude quaternion of the spacecraft and $\bar{\mathbf{q}}$ be the desired attitude quaternion. The attitude error in terms of quaternion is defined as [4],

$$\delta\mathbf{q} \equiv \begin{bmatrix} \delta\mathbf{q}_{1:3} \\ \delta q_4 \end{bmatrix} = \mathbf{q} \otimes \bar{\mathbf{q}}^{-1} = \begin{bmatrix} \boldsymbol{\Xi}(\bar{\mathbf{q}}^{-1}) & \bar{\mathbf{q}}^{-1} \end{bmatrix} \mathbf{q} \tag{5}$$

where,

$$\boldsymbol{\Xi}(\bar{\mathbf{q}}^{-1}) = \begin{bmatrix} \bar{q}_4 & \bar{q}_3 & -\bar{q}_2 \\ -\bar{q}_3 & \bar{q}_4 & \bar{q}_1 \\ \bar{q}_2 & -\bar{q}_1 & \bar{q}_4 \\ \bar{q}_1 & \bar{q}_2 & \bar{q}_3 \end{bmatrix}. \tag{6}$$

Let the constraints on angular velocity components in rad/sec be described as follows

$$|\omega_1(t)| \le k_1, |\omega_2(t)| \le k_2, \ |\omega_3(t)| \le k_3 \tag{7}$$

The main adaptive output feedback control law is proposed as

$$\boldsymbol{u} = -\hat{\mathbf{I}}\mathbf{K}_v^{-1}(\delta\mathbf{q}_{1:3} + k_5\mathbf{v}), \tag{8}$$

where, $\mathbf{v} = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}^T$ is the synthesized angular velocity to be introduced later, and

$$\mathbf{K}_v = \begin{bmatrix} \frac{k_4}{k_1^2 - v_1^2} & 0 & 0 \\ 0 & \frac{k_4}{k_2^2 - v_2^2} & 0 \\ 0 & 0 & \frac{k_4}{k_3^2 - v_3^2} \end{bmatrix} \tag{9}$$

and $\hat{\mathbf{I}}$ is the estimated value of inertia matrix $\mathbf{I}$. Let $\mathbf{J} = \mathbf{I}^{-1}$, $\Delta\hat{\mathbf{J}} - \mathbf{J}$ and define

$$\boldsymbol{\Theta} := \begin{bmatrix} \Delta \mathrm{J}_{11} & \Delta \mathrm{J}_{12} & \Delta \mathrm{J}_{13} & \Delta \mathrm{J}_{22} & \Delta \mathrm{J}_{23} & \Delta \mathrm{J}_{33} \end{bmatrix}^T \tag{10}$$

then, adaptation law for $\mathbf{I}$ is introduced as

$$\dot{\hat{\mathbf{I}}} = -\hat{\mathbf{I}}\dot{\hat{\mathbf{J}}}\hat{\mathbf{I}} \tag{11}$$

and

$$\dot{\mathbf{\Theta}} = -\mathbf{\Gamma M}^T \hat{\mathbf{I}} \mathbf{K}_v^{-1} (\delta \mathbf{q}_{1:3} + K_5 \mathbf{V}) \quad (12)$$

where,

$$\mathbf{M} = \begin{bmatrix} y_1 & y_2 & y_3 & 0 & 0 & 0 \\ 0 & y_1 & 0 & y_2 & y_3 & 0 \\ 0 & 0 & y_1 & 0 & y_2 & y_3 \end{bmatrix} \quad (13)$$

where,

$$\mathbf{y} = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix}^T = \mathbf{K}_v \mathbf{v} \quad (14)$$

For a discussion on the convergence of the Eq. (12) the reader may refer to Ref. [20] among other references.

One advantage of control law (8) lies in the fact that it is an output feedback control law, i.e. there is no need to measure angular velocity of the rigid spacecraft and this is achieved by using the synthesized angular velocity instead of measured angular velocity. A procedure similar to that presented in Ref. [11] can be followed to construct synthesized angular velocity. The synthesized angular velocity is defined as

$$\mathbf{v} = 2\boldsymbol{\Xi}^T(\delta\mathbf{q})z \quad (15)$$

where, z is obtained by passing $\delta\dot{\mathbf{q}}$ through an LTI strictly proper and strictly positive real system $\mathbf{C}(s)$

$$z = \mathbf{C}(s)\,\delta\dot{\mathbf{q}} \quad (16)$$

To design and implement this filter consider a minimal realization of $\mathbf{C}(s)$ as

$$\dot{\boldsymbol{\xi}} = \boldsymbol{A}\boldsymbol{\xi} + \boldsymbol{B}\delta\dot{\mathbf{q}};\; z = \boldsymbol{C}\boldsymbol{\xi} \quad (17)$$

Since $\mathbf{C}(s)$ is strictly positive real and strictly proper, the Kalman–Yakubovich–Popov's Lemma implies that there exist positive definite matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$ such that

$$\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{PA} = -\boldsymbol{Q};\quad \boldsymbol{PB} = \boldsymbol{C}^T \quad (18)$$

and (15) is implementable by choosing any Hurwitz matrix $\boldsymbol{A}$, full column rank matrix $\boldsymbol{B}$ and positive definite matrix $\boldsymbol{Q}$. Define the state variable $\boldsymbol{\xi}_1$ as $\dot{\boldsymbol{\xi}}_1 = \boldsymbol{\xi}$. Taking the derivative of both sides of (17) leads to

$$\begin{aligned} \dot{\boldsymbol{\xi}}_1 &= \boldsymbol{A}\boldsymbol{\xi}_1 + \boldsymbol{B}\delta\mathbf{q} \\ z &= \boldsymbol{C}\dot{\boldsymbol{\xi}}_1 = \boldsymbol{B}^T\boldsymbol{P}(\boldsymbol{A}\boldsymbol{\xi}_1 + \boldsymbol{B}\delta\mathbf{q}) \end{aligned} \quad (19)$$

The above results are summarized in the following theorem and the rigorous stability analysis is presented.

Theorem 1 *The control law (8) almost globally stabilizes the system described by (2) and (4).*

Proof The closed loop system equation can be written as

$$\mathbf{I}\dot{\boldsymbol{\omega}} = -[\boldsymbol{\omega}\times]\mathbf{I}\boldsymbol{\omega} - \mathbf{I}\mathbf{K}_v^{-1}(\delta\mathbf{q}_{1:3} + k_5\boldsymbol{\omega}) \quad (20)$$

and the equilibrium points of this system are

$$(\delta\mathbf{q}, \boldsymbol{\omega}) = \left(\left[\delta\mathbf{q}_{1:3}; \delta q_4\right], \boldsymbol{\omega}\right) = ([\mathbf{0}; \pm 1], \mathbf{0}). \quad (21)$$

Both of these equilibrium points are associated with one physical attitude. Consider the Lyapunov function

$$V = (1 - \delta q_4)^2 + \delta\mathbf{q}_{1:3}^T\delta\mathbf{q}_{1:3} + \frac{1}{2}k_4\sum_{i=1}^{3}\ln\frac{k_i^2}{k_i^2 - \omega_i^2} > 0 \quad (22)$$

The first two terms in this Lyapunov function are a measure of potential energy of the rigid body w.r.t reference attitude and the logarithmic term was first proposed in Ref. [31] to treat constraints on the angular velocity and has been also used in Ref. [18]. Taking time derivative of this Lyapunov function yields

$$\dot{V} = -2\left(1 - \delta q_4\right)\delta\dot{q}_4 + 2\delta\mathbf{q}_{1:3}^T\,\delta\dot{\mathbf{q}}_{1:3} + \boldsymbol{\omega}^T\mathbf{K}_{\omega}\dot{\boldsymbol{\omega}} \quad (23)$$

by substituting for $\delta\dot{\mathbf{q}}$ and $\dot{\boldsymbol{\omega}}$, (23) reduces to

$$\dot{V} = -k_5\boldsymbol{\omega}^T\boldsymbol{\omega} - \boldsymbol{\omega}^T\mathbf{K}_{\omega}\mathbf{I}^{-1}[\boldsymbol{\omega}\times]\mathbf{I}\boldsymbol{\omega} \quad (24)$$

for investigating the sign of $\dot{V}$, let define

$$\mathbf{G} = \mathbf{K}_{\omega}\mathbf{I}^{-1}[\boldsymbol{\omega}\times]\mathbf{I} \quad (25)$$

which leads to

$$\dot{V} = -k_5\boldsymbol{\omega}^T\boldsymbol{\omega} - \boldsymbol{\omega}^T\mathbf{G}\boldsymbol{\omega} \quad (26)$$

Let the upper bound on the Euclidean norm $\|\mathbf{G}\|$ be known, then choosing $k_5 > ||\mathbf{G}||$ leads to

$$\dot{V} \leq 0. \quad (27)$$

It means $\dot{V}$ is negative semi definite and hence the equilibrium points $([\mathbf{0}; \pm 1], \mathbf{0})$ are stable. To prove asymptotic stability, we use Lasalle theorem. As the system is stable it yields that $\delta\mathbf{q}, \boldsymbol{\omega} \in \mathbf{L}_\infty$. Taking integral of both sides of equation (26) (e.g. for $k_5 = ||\mathbf{G}|| + 1$) yields $\boldsymbol{\omega} \in \mathbf{L}_2$ and hence $\boldsymbol{\omega} \in \mathbf{L}_\infty \cap \mathbf{L}_2$. In the other hand from (20), it can be concluded that $\dot{\boldsymbol{\omega}} \in \mathbf{L}_\infty$. Then using Barbalat's Lemma we have $\lim_{t\to\infty}\boldsymbol{\omega} = 0$. The equation of closed loop system (20) shows that $\lim_{t\to\infty}\boldsymbol{\omega} = 0$ only if $\lim_{t\to\infty}\delta\mathbf{q}_{1:3} = 0$. Hence

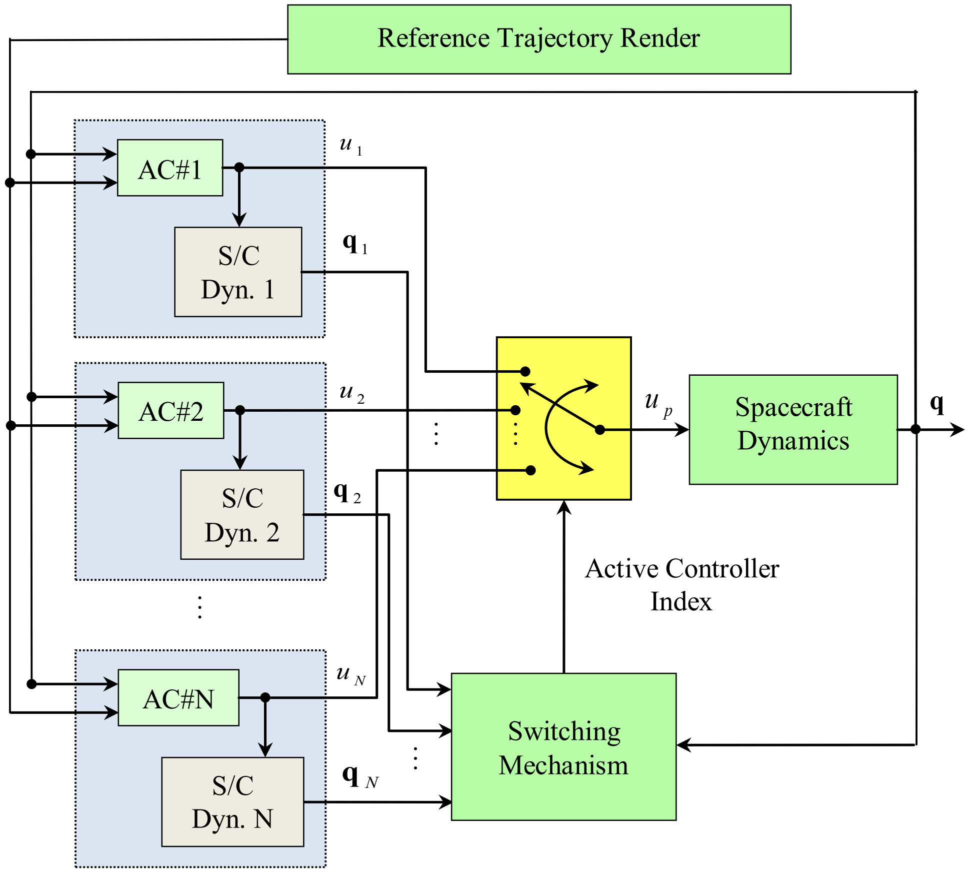


Fig. 1 Structure of the proposed multiple model adaptive attitude controller

the largest invariant subset in $\Omega = \{(\delta\mathbf{q}, \boldsymbol{\omega}) | \dot{V}(x) = 0\}$ is $\{([\delta\mathbf{q}_{1:3}; \delta q_4], \boldsymbol{\omega}) = ([0; 1], 0)\}$. So the asymptotic stability is proved by Lasalle theorem. □

It should be mentioned that the proposed attitude control law does not guarantee the shortest path to be travelled. The stability analysis in Ref. [20] is not rigorous as claimed, e.g. it is not mentioned whether the controller globally stabilizes the system or not. This seems to be because of the above-mentioned ambiguity in stability analysis of control systems in terms of quaternion coordinate.

The next step is to apply multiple model and switching approach to the main control law (8). Usually there are two possibilities for generating model bank in multiple model adaptive control scheme. First one is generating models based on system dynamics described in various coordinates or models obtained by different simplification methods. Second possibility is keeping one governing dynamics equation and establishing models by dividing parameter space of the plant. In this paper the latter choice is adopted by considering the parameter space of inertia matrix $\mathbf{I} \in \Re^{3\times3}$. Since the inertia matrix is symmetric, the parameter space is

$$\boldsymbol{S} = \left\{ \boldsymbol{\theta} \in \Re^6 \middle| \mathbf{I} \succ 0, \ell_i \leq \theta_i \leq u_i, \quad i = 1, 2, \ldots, 6 \right\} \quad (28)$$

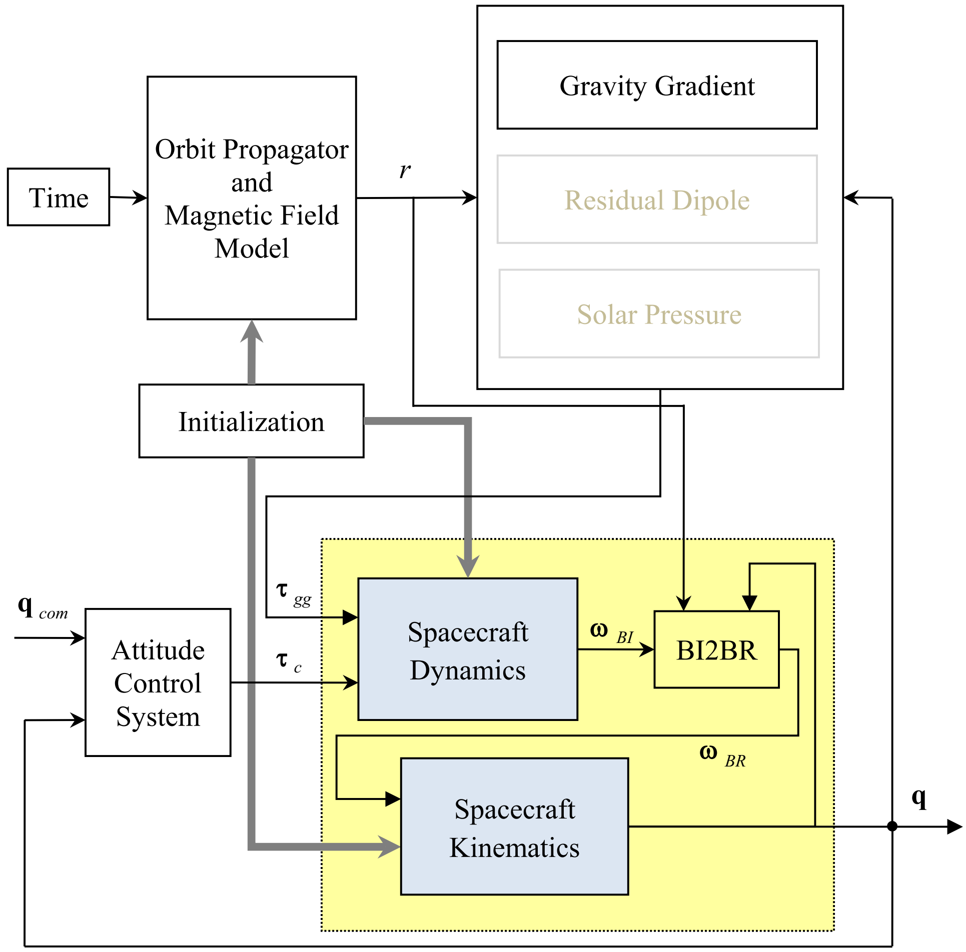
where the constraints $\ell_i \leq \theta_i \leq u_i$, $i = 1, 2, \ldots, 6$ are known based on a priori knowledge of the inertia matrix entries. This parameter space can be broken to N subspaces by considering the N initial choices for the inertia matrix in main adaptive law. Choosing N depends on the trade-off between desire performance and controller complexity made by the designer. The actual inertia matrix and other initial choices are named as $\mathbf{I}_p$: actual inertia matrix of the spacecraft; $\mathbf{I}_i$, $i = 1, 2, \ldots, N$: inertia matrix choices with different amounts of uncertainty corresponding to different subspaces in $\boldsymbol{S}$.

The structure of the proposed multiple model adaptive attitude control is shown in Fig. 1. In this figure N identification models are constructed by the N initial choices for inertia matrix using the parameter adaptation law (11), and spacecraft attitude dynamics and kinematics Eqs. (2) and (4).

In multiple model adaptive control, switching mechanism is an essential part which determines active controller at every instance based on some measured signals and identified models. The proposed switching mechanism selects the nearest model–controller pair to the actual plant based on the following criteria:

$$\text{active controller index} = \underset{i=1,2,\ldots,N}{\text{argmin}} \left\| \mathbf{q}_p - \mathbf{q}_i \right\|_2 \quad (29)$$

Fig. 2 Block diagram of the LEO satellite attitude simulator



where $\mathbf{q}_p$ is the quaternion vector representation for the actual attitude of the plant.

4 Simulation Results on a LEO Satellite Simulator

This section explains the specification of the developed attitude simulator for a typical LEO satellite which is used to demonstrate the efficacy of the proposed control algorithm.

4.1 LEO Satellite Simulator

The block diagram of this simulator is shown in Fig. 2.

4.1.1 Coordinate Frames

To determine the position and attitude of a spacecraft in space it is needed to define at least two reference frames, and more are needed for convenience. Spacecraft body frame and an inertial reference frame are necessary and an intermediate reference frame such as orbit reference frame is used for convenient transformation.

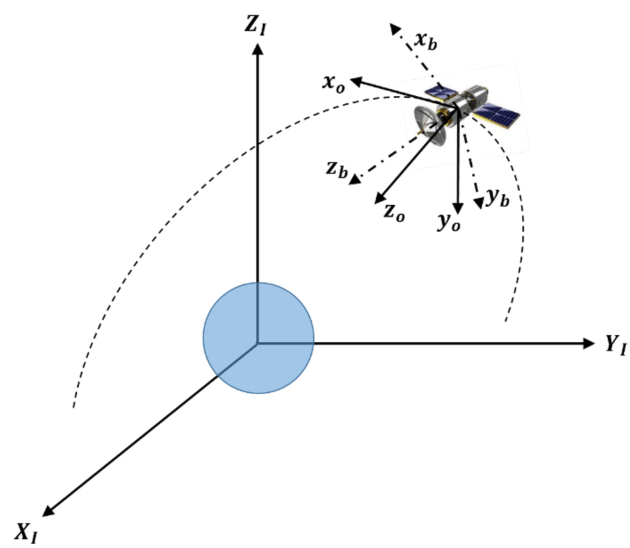


Fig. 3 Illustration of coordinate frames

(a) Spacecraft body frame: the origin of spacecraft body frame is located at the center of mass of spacecraft and its axes are a triad of orthogonal right-hand axes. In the case that

Table 1 The parameters value used in simulation

Parameter	Value
Orbit parameters	
Height	800 km
Inclination	$\pi/3$ rad
Right ascension	0
Argument of perigee	0
Mean anomaly	0
Satellite actual inertia matrix	$\mathbf{I}_p = \begin{bmatrix} 300 & 50 & 20 \\ 50 & 110 & 0 \\ 20 & 0 & 800 \end{bmatrix}$
Controller 1 Initial inertia matrix	$\mathbf{I}_1 = \begin{bmatrix} 15 & 3 & 2 \\ 3 & 50 & 4 \\ 2 & 4 & 14 \end{bmatrix}$
Controller 2 Initial inertia matrix	$\mathbf{I}_2 = \begin{bmatrix} 250 & 30 & 20 \\ 30 & 90 & 4 \\ 20 & 4 & 500 \end{bmatrix}$
Constant gains of main controller	
k_1	10
k_2	10
k_3	10
k_4	12.5
k_5	0.8
$\mathbf{0}$	$0.05 \times \text{eye}(6)$
Lead filter realization	
$\mathbf{A}$	$-30 \times \text{eye}(4)$
$\mathbf{B}$	$33 \times \text{eye}(4)$
$\mathbf{Q}$	$30 \times \text{eye}(4)$
$\mathbf{C}$	$16.5 \times \text{eye}(4)$
Initial quaternion	$(0, 0, 0, -1)$
Commanded quaternion	$(-0.2448, -0.1821, -0.3676, -0.8785)$
Initial Euler angles	(0, 0, 0) degrees
Commanded Euler angles	(10, 30, 50) deg
Simulation step size	0.05 s
Solver	Runge–Kutta

body frame axes coincide with principal moment of inertia axes, spacecraft inertia matrix will be diagonal.

(b) Earth Centered Inertia (ECI) frame: a frame in which Newton's law of motion hold is called an inertial frame. For engineering purposes, a frame with fixed direction relative to the solar system can be considered as inertial frame. The origin of this reference frame is located at the center of mass of the earth and its X axis is in the direction of vernal equinox vector and the Z axis coincides with the positive axis of the rotation of earth. The equatorial plane is the X–Y plane of this frame and the Y axis is defined such that it completes a right-hand orthogonal triad with X and Y axis.

(c) Orbit reference frame: this reference frame is used as an intermediate frame and its origin is located at the center of mass of spacecraft. The Z axis points toward earth center of mass from spacecraft center of mass (nadir direction), the X axis is perpendicular to the Z axis in the orbital plane and in the direction of spacecraft velocity. The Y axis completes a right-hand orthogonal system. These three reference frames are illustrated in Fig. 3.

4.1.2 Orbit Propagator

Since our focus is on attitude dynamics we develop a simple circular Keplerian orbit propagator. Numerical propagation of spacecraft orbit can be carried out using classical Keplerian elements or position and velocity vectors. Based on the two-body problem and Newton's universal law of gravitation, the equation of motion of a Keplerian orbit can be written as [4]

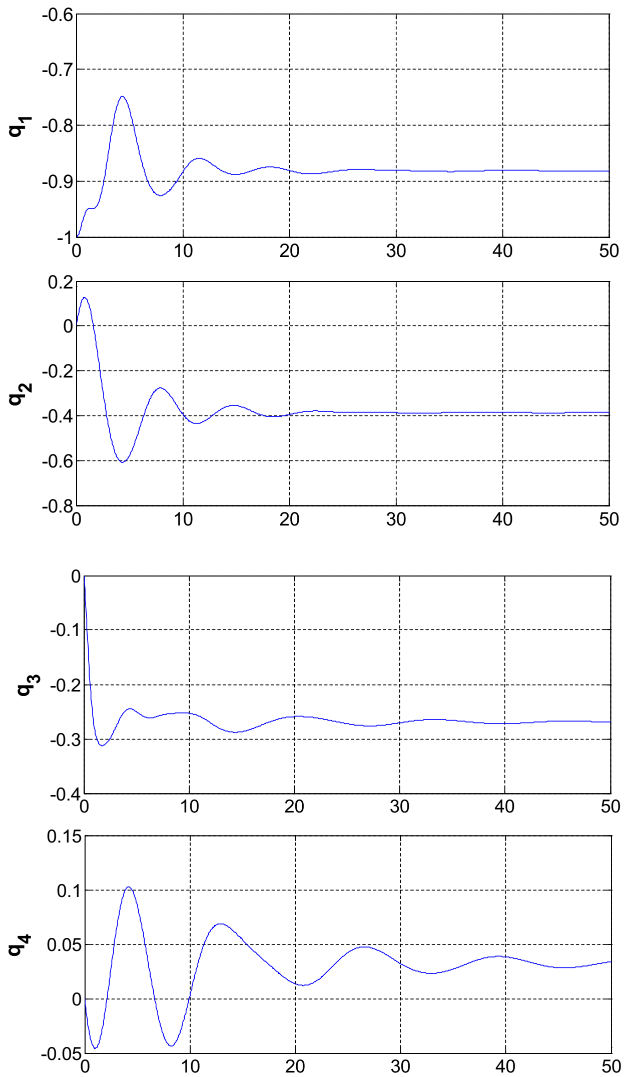


Fig. 4 Quaternion vector vs time (s) for pure adaptive controller

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} = -\mu \frac{\mathbf{r}}{r^3} \tag{30}$$

where, $\mu = G(M + m) \approx GM^{3\times3}$ is the gravitational parameter, M and m are earth and spacecraft mass respectively and G is Newton's universal gravitational constant.

4.1.3 Spacecraft Kinematics and Dynamics

Spacecraft kinematics and dynamics are implemented in simulator according to (2) and (4) described in Sect. 2. A point that should be mentioned is about angular velocity transformation. As the Euler moment equation is written in body frame the dynamics equation is in term of $\boldsymbol{\omega}_{BI-B}$ which is the angular velocity of body frame with respect to inertial frame expressed in body frame. While the kinematics equation is in term of $\boldsymbol{\omega}_{BR-B}$ which is the angular velocity of body

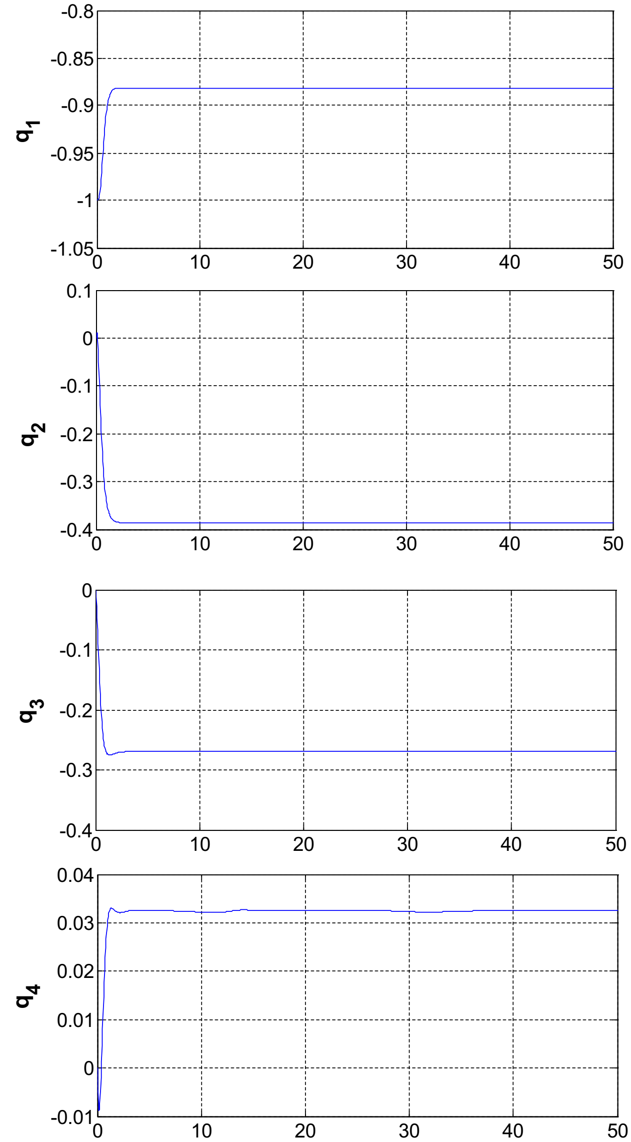


Fig. 5 Quaternion vs time (s) for multiple model adaptive controller

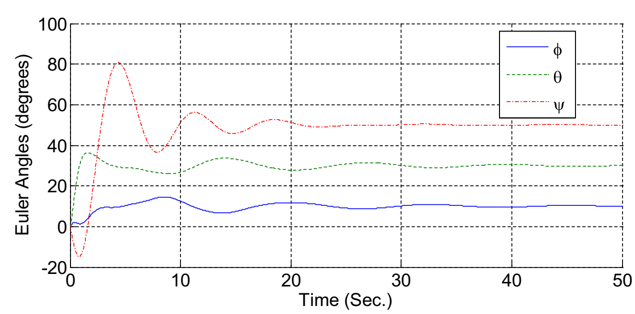


Fig. 6 Euler angles step response for pure adaptive controller

frame with respect to the orbital reference frame expressed in body frame and hence $\boldsymbol{\omega}_{BI-B}$ should be properly transformed to $\boldsymbol{\omega}_{BR-B}$. Let $\boldsymbol{\omega}_0$ be orbital angular velocity, this

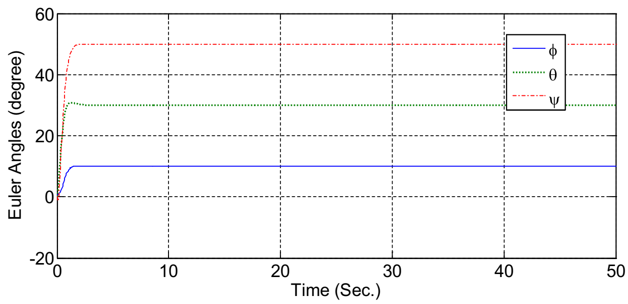


Fig. 7 Euler angles step response for multiple model adaptive controller

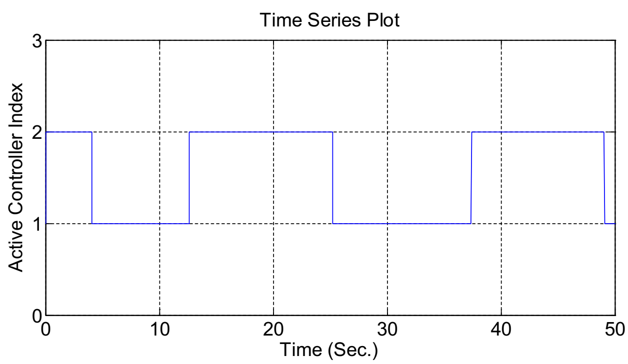


Fig. 8 Active controller index in multiple model control

transformation is (see Ref. [3])

$$\boldsymbol{\omega}_{\mathrm{BR-B}} = \boldsymbol{\omega}_{\mathrm{BI-B}} - [\mathbf{A}_{RB}] \begin{bmatrix} 0 \\ -\omega_0 \\ 0 \end{bmatrix} \tag{31}$$

where, $[\boldsymbol{A}_{RB}]$ is the direction cosine matrix which transforms a vector in reference frame to body frame and can be computed from the quaternion during simulation.

4.1.4 Disturbance Torques

There are various sources of disturbance torques acting on spacecraft such as gravity gradient torque, solar pressure, and residual dipole. In this simulator we consider gravity gradient due to its major influence on spacecraft dynamics. An adequate approximation of gravity gradient torque is given by Markley [4] as

$$\boldsymbol{\tau}_{GG} = \frac{3\mu}{r^3}\mathbf{n} \times (\mathbf{In}) \tag{32}$$

where $\mathbf{n}$ is a unit vector in direction of nadir expressed in body frame which is in turn

$$\mathbf{n} = [\mathbf{A}_{RB}] \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \tag{33}$$

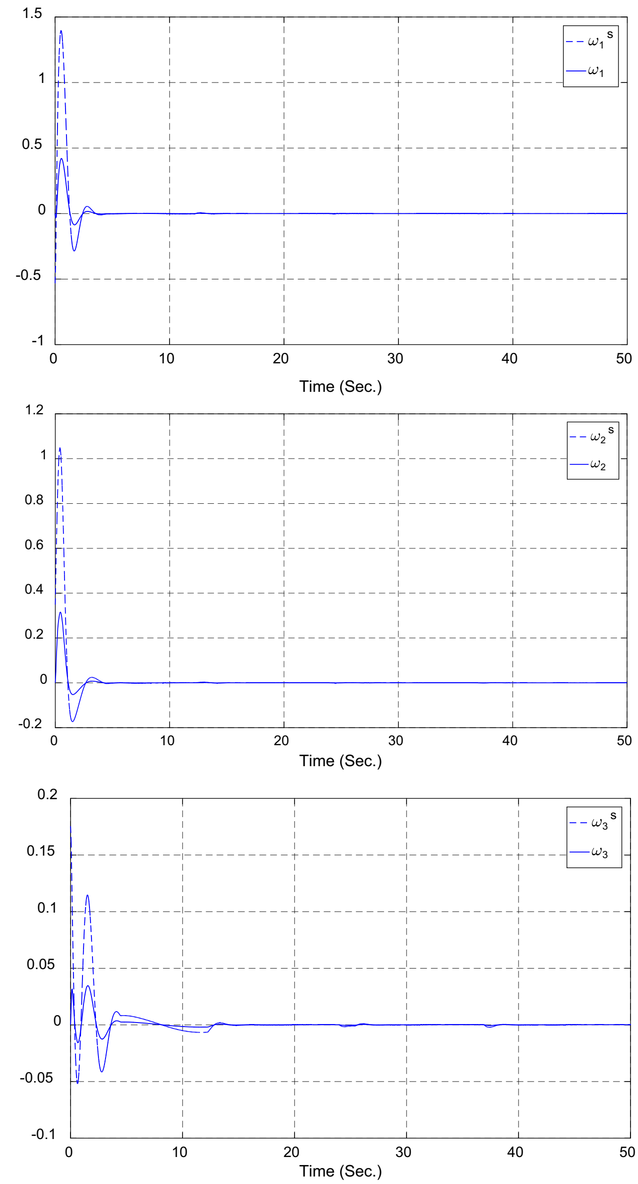


Fig. 9 Comparison of angular velocity components and pseudo-angular velocity components

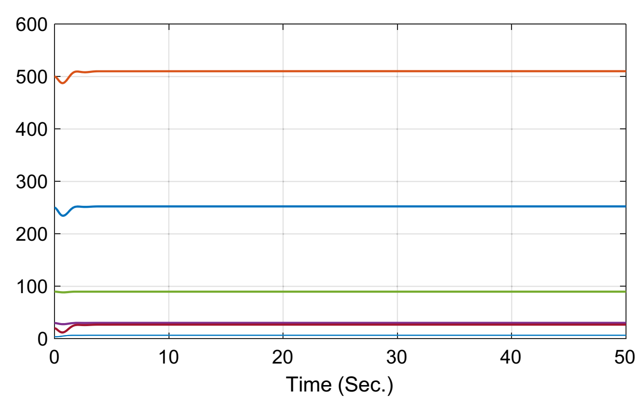


Fig. 10 Adaptation of components of the inertia matrix

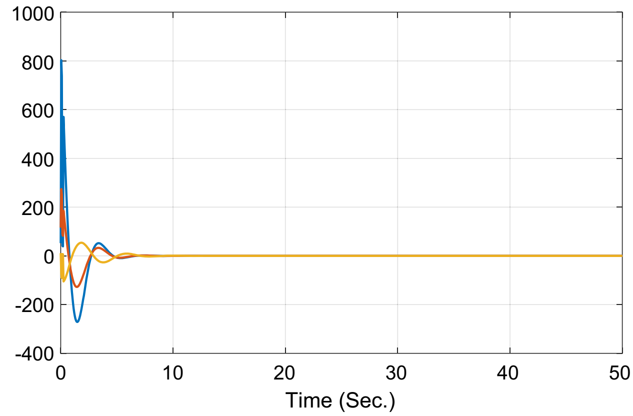


Fig. 11 Time history of input signals

4.2 Multiple Model Adaptive Attitude Controller Simulation

In this section, the proposed multiple model adaptive control scheme is applied to the developed simulator. We consider $N = 2$ which means we have two identification models. To evaluate the performance of the proposed control scheme, the problem of regulating the spacecraft attitude to a fixed commanded attitude is investigated. Since Euler angles are more intuitive, system response in terms of Euler angles is also shown and thus appropriate transformation is used. The values of parameters used in simulation are given in Table 1.

Simulation results for both, the pure adaptive control and the multiple model adaptive control are shown in Figs. 4, 5, 6, 7 and 8. Figure 4 shows quaternion behavior for the pure adaptive control and Fig. 5 shows that of multiple model control. Although in both cases the quaternion vector converges to the commanded quaternion, the performance improvement due to incorporating multiple model approach can be clearly seen in Fig. 5.

Figure 8 shows the time histories of switching between two controllers. For the proposed switching logic, switching between controllers will not stop even after convergence to the commanded quaternion. This may in part be because of disturbance effect and hence one could invoke sort of feedforward terms in control law to avoid unwanted switching. Modifying the switching logic by insertion of dwell time or hysteresis is also possible.

Figure 9 depicts the convergence of the components of the pseudo velocity to those of the actual angular velocity. For all three components, the pseudo velocity component converges to the actual value after a few seconds. As shown in Fig. 10, adaptation of the inertia matrix converges to the final values fast. However, since the persistent excitation (PE) condition of the input signal is not met, the final converged values are not the actual values of the components of the inertia matrix. The input signal is also shown in Fig. 11.

5 Conclusion

In this work, the output feedback variant of the adaptive attitude controller under angular velocity constraints proposed in Ref. [20] was derived. This output feedback adaptive controller was then considered as a main control law and its transient response for the regulation problem was improved significantly by applying the multiple model and switching approach to adaptive control. In particular, the multiple model and switching approach was achieved by dividing the parameter space of inertia matrix to smaller subspaces. A compact LEO satellite attitude simulator was developed and used for evaluation of the proposed control scheme. Future works includes investigation of robustness of control law (8) against bounded time variant external disturbances and reconsidering the same problem taking into account actuator saturation.

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