The 25th Iranian Conference on Electrical Engineering
(ICEE2017)

Adaptive Output Feedback Attitude Control of a LEO Satellite under Angular Velocity Constraints

Abolfazl Shahrooei, Mohammad Hosein Kazemi

Department of Control Engineering
Shahed University

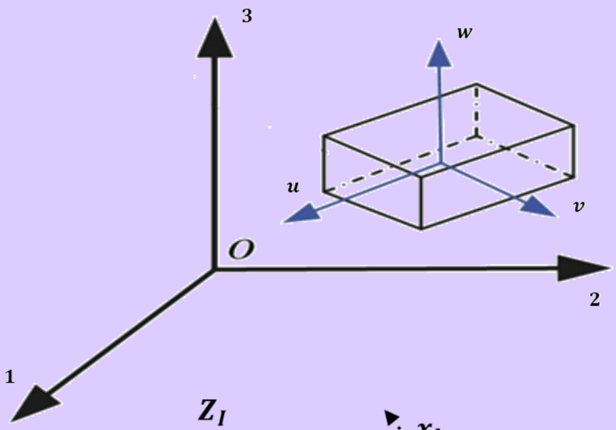
May 2017

Outline

- Introduction
- Mathematical Model of Spacecraft Attitude
- The Proposed Control Scheme
- Simulation Results
- Conclusion

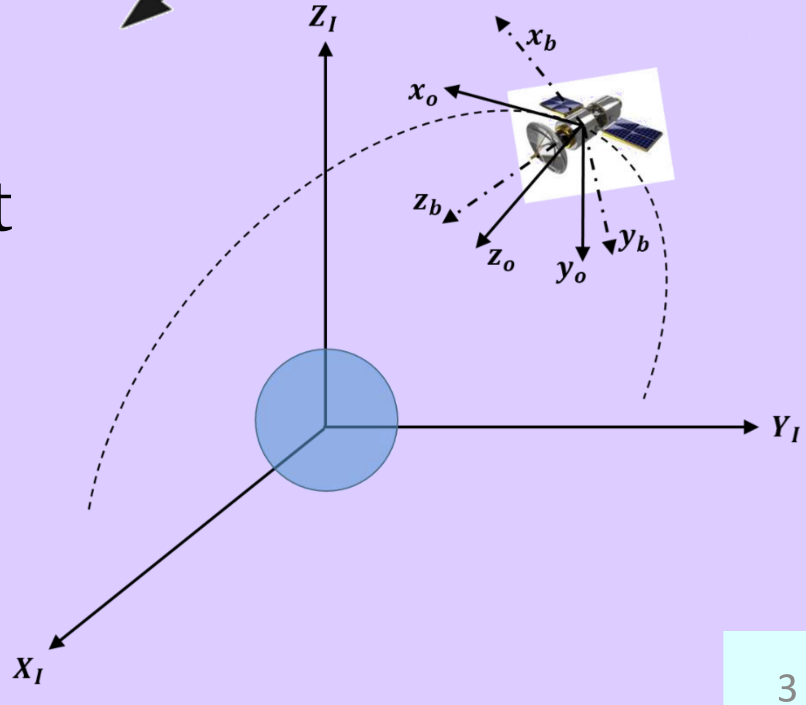
Introduction

- Definition of Attitude
  - Concept
  - Formal definition



- The Attitude Control Problem

- Attitude Control System (ACS) of a Spacecraft



Introduction

- Texts:
  - (Wertz, 2012), (Schaub & Junkins, 2003), (Sidi, 1997), (Markley & Crassidis, 2014)
- Works on Stabilizing
  - (Wie & Barba, 1985), (Tsiotras,1996),
- Output Feedback controllers
  - (Salcudean, 1991), (Lizarralde &Wen, 1996), (Zhou, 2014), (Tsiotras 1998),
- Woks on Adaptive Attitude Control
  - (Schaub et. al 2001), (Singla et. al, 2006)
- Attitude Control under Angular Velocity Constraints
  - (Singla & Tarunraj, 2008), (Ngo, et.al 2004)

Mathematical Model

- Rotational Motion:
  - Kinematics
  - Dynamics
- Attitude Representation Parameters:
  - Direction Cosine Matrix
  - Euler Angles
  - Quaternion
  - Rodrigues Parameters
  - Modified Rodrigues Parameters

Mathematical Model

The quaternion vector representing the attitude of body frame w.r.to inertial frame:

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_{1:3} \\ q_4 \end{bmatrix} \tag{1}$$

Kinematics:

$$\dot{\mathbf{q}} = \frac{1}{2}\mathbf{\Omega}(\boldsymbol{\omega}) \tag{2}$$

where

$$\mathbf{\Omega}(\boldsymbol{\omega}) = \begin{bmatrix} -[\boldsymbol{\omega}\times] & \boldsymbol{\omega} \\ -\boldsymbol{\omega}^{\mathbf{T}} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \tag{3}$$

Euler's moment equation gives the nonlinear three-axis dynamics equation of a rigid body as

$$\mathbf{I}\dot{\boldsymbol{\omega}} = -[\boldsymbol{\omega}\times]\mathbf{I}\boldsymbol{\omega} + \mathbf{u} \tag{4}$$

where $\mathbf{I} \in \mathbb{R}^{\mathbf{3}\times\mathbf{3}}$, $\mathbf{u} \in \mathbb{R}^{\mathbf{3}}$.

The Proposed Control Scheme

Attitude error in terms of quaternion:

$$\delta\mathbf{q} \equiv \begin{bmatrix} \delta\mathbf{q}_{1:3} \\ \delta q_4 \end{bmatrix} = \mathbf{q} \otimes \overline{\mathbf{q}}^{-1} = [\boldsymbol{\Xi}(\overline{\mathbf{q}}^{-1}) \quad \overline{\mathbf{q}}^{-1}]\mathbf{q} \tag{5}$$

where

$$\boldsymbol{\Xi}(\overline{\mathbf{q}}^{-1}) = \begin{bmatrix} \bar{q}_4 & \bar{q}_3 & -\bar{q}_2 \\ -\bar{q}_3 & \bar{q}_4 & \bar{q}_1 \\ \bar{q}_2 & -\bar{q}_1 & \bar{q}_4 \\ \bar{q}_1 & \bar{q}_2 & \bar{q}_3 \end{bmatrix} \tag{6}$$

constraints on angular velocity components:

$$|\omega_1(t)| \leq k_1 , |\omega_2(t)| \leq k_2, |\omega_3(t)| \leq k_3 \tag{7}$$

The output feedback control law is proposed as

$$\mathbf{u} = -\mathbf{I}\mathbf{K}_v^{-1}(\delta\mathbf{q}_{1:3} + k_5\mathbf{v}) \tag{8}$$

where $\mathbf{v} = [v_1 \quad v_2 \quad v_3]^T$ is the pseudo-angular velocity, and

$$\mathbf{K}_v = \begin{bmatrix} \frac{k_4}{k_1^2 - v_1^2} & 0 & 0 \\ 0 & \frac{k_4}{k_2^2 - v_2^2} & 0 \\ 0 & 0 & \frac{k_4}{k_3^2 - v_3^2} \end{bmatrix} \tag{9}$$

The Proposed Control Scheme

Pseudo-velocity:

following a procedure similar to that presented in (Tsiotras 1998), pseudo-velocity is defined as

$$\mathbf{v} = 2\boldsymbol{\Xi}^{\mathrm{T}}(\delta\mathbf{q})\mathbf{z} \tag{10}$$

where $\mathbf{z}$ is obtained by passing $\delta\dot{\mathbf{q}}$ through an LTI strictly proper and strictly positive real system $\mathbf{C}(s)$

$$\mathbf{z} = \mathbf{C}(s)\delta\dot{\mathbf{q}} \tag{11}$$

consider a minimal realization of $\mathbf{C}(s)$ as

$$\dot{\boldsymbol{\xi}} = \mathbf{A}\boldsymbol{\xi} + \mathbf{B}\delta\dot{\mathbf{q}}; \;\; \mathbf{z} = \mathbf{C}\boldsymbol{\xi}. \tag{12}$$

since $\mathbf{C}(s)$ is strictly positive real and strictly proper, the Kalman-Yakubovich-Popov's Lemma implies that there exist positive definite matrices $\mathbf{P}$ And $\mathbf{Q}$ Such that

$$\mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A} = -\mathbf{Q}; \qquad \mathbf{P}\mathbf{B} = \mathbf{C}^{\mathrm{T}} \tag{13}$$

by choosing any Hurwitz matrix $\mathbf{A}$, full column rank matrix $\mathbf{B}$, and positive definite matrix $\mathbf{Q}$, we have

$$\begin{gathered} \dot{\boldsymbol{\xi}}_1 = \mathbf{A}\boldsymbol{\xi}_1 + \mathbf{B}\delta\mathbf{q}, \\ \mathbf{z} = \mathbf{C}\dot{\boldsymbol{\xi}}_1 = \mathbf{B}^{\mathrm{T}}\mathbf{P}(\mathbf{A}\boldsymbol{\xi}_1 + \mathbf{B}\delta\mathbf{q}) \end{gathered} \tag{14}$$

The Proposed Control Scheme

Theorem: The system described by

$$\dot{\mathbf{q}} = \frac{1}{2}\boldsymbol{\Omega}(\boldsymbol{\omega})$$

$$\mathbf{I}\dot{\boldsymbol{\omega}} = -[\boldsymbol{\omega}\times]\mathbf{I}\boldsymbol{\omega} + \mathbf{u}$$

Will be almost globally stabilized by control law

$$\mathbf{u} = -\mathbf{I}\mathbf{K}_v^{-1}(\delta\mathbf{q}_{1:3} + k_5\boldsymbol{\omega})$$

Proof:

Closed loop system: $\mathbf{I}\dot{\boldsymbol{\omega}} = -[\boldsymbol{\omega}\times]\mathbf{I}\boldsymbol{\omega} - \mathbf{I}\mathbf{K}_v^{-1}(\delta\mathbf{q}_{1:3} + k_5\boldsymbol{\omega})$

Equilibrium point: $(\delta\mathbf{q}, \boldsymbol{\omega}) = ([\delta\mathbf{q}_{1:3}; \delta q_4], \boldsymbol{\omega}) = ([\mathbf{0}; 1], \mathbf{0})$

$$V = 2(1-\delta q_4) + \frac{1}{2}k_4\sum_{i=1}^{3}\ln\frac{k_i^2}{k_i^2-\omega_i^2} = (1-\delta q_4)^2 + \delta\mathbf{q}_{1:3}^T\delta\mathbf{q}_{1:3} + \frac{1}{2}k_4\sum_{i=1}^{3}\ln\frac{k_i^2}{k_i^2-\omega_i^2} > 0$$

$$\dot{V} = -2(1-\delta q_4)\delta\dot{q}_4 + 2\delta\mathbf{q}_{1:3}^T\delta\dot{\mathbf{q}}_{1:3} + \boldsymbol{\omega}^T\mathbf{K}_{\boldsymbol{\omega}}\dot{\boldsymbol{\omega}}$$

The Proposed Control Scheme

Proof: continued

$$\delta\dot{\mathbf{q}} = \frac{1}{2}\mathbf{\Omega}(\boldsymbol{\omega})\delta\mathbf{q}$$

$$\dot{\mathrm{V}} = -\mathrm{k}_5\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\omega} - \boldsymbol{\omega}^{\mathrm{T}}\mathbf{K}_{\boldsymbol{\omega}}\mathbf{I}^{-1}[\boldsymbol{\omega}\,\times]\mathbf{I}\boldsymbol{\omega}$$

Let define $\mathbf{G} = \mathbf{K}_{\boldsymbol{\omega}}\mathbf{I}^{-1}[\boldsymbol{\omega}\,\times]\mathbf{I}$ then

$$\dot{\mathrm{V}} = -\mathrm{k}_5\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\omega} - \boldsymbol{\omega}^{\mathrm{T}}\mathbf{G}\boldsymbol{\omega}$$

let the upper bound on the Euclidean norm of $\mathbf{G}$ be known as

$$\|\mathbf{G}\| < \mathrm{g}$$

choosing $\mathrm{k}_5 > \mathrm{g}$ Leads to

$$\dot{\mathrm{V}} \le 0$$

i.e. $\dot{\mathrm{V}}$ is negative semi definite And hence the equilibrium point $([\mathbf{0}; 1], \mathbf{0})$ is stable.

As the system is stable it yields that $\delta\mathbf{q}, \boldsymbol{\omega} \in \mathbf{L}_\infty$. Taking integral of both sides of above equation yields $\boldsymbol{\omega} \in \mathbf{L}_2$ and hence $\boldsymbol{\omega} \in \mathbf{L}_\infty \cap \mathbf{L}_2$. by using Barbalat's Lemma we have $\lim_{t\to\infty} \boldsymbol{\omega} = 0$. The equation of closed loop system shows that $\lim_{t\to\infty} \boldsymbol{\omega} = 0$ only if $\lim_{t\to\infty} \delta\mathbf{q}_{1:3} = 0$. Hence the largest invariant subset in $\Omega = \left\{(\delta\mathbf{q}, \boldsymbol{\omega}) \middle| \dot{\mathrm{V}}(\mathrm{x}) = 0\right\}$ is$\{([\delta\mathbf{q}_{1:3}; \delta\mathrm{q}_4], \boldsymbol{\omega}) = ([\mathbf{0}; 1], \mathbf{0})\}$. So the asymptotic stability is proved by Lasalle theorem.

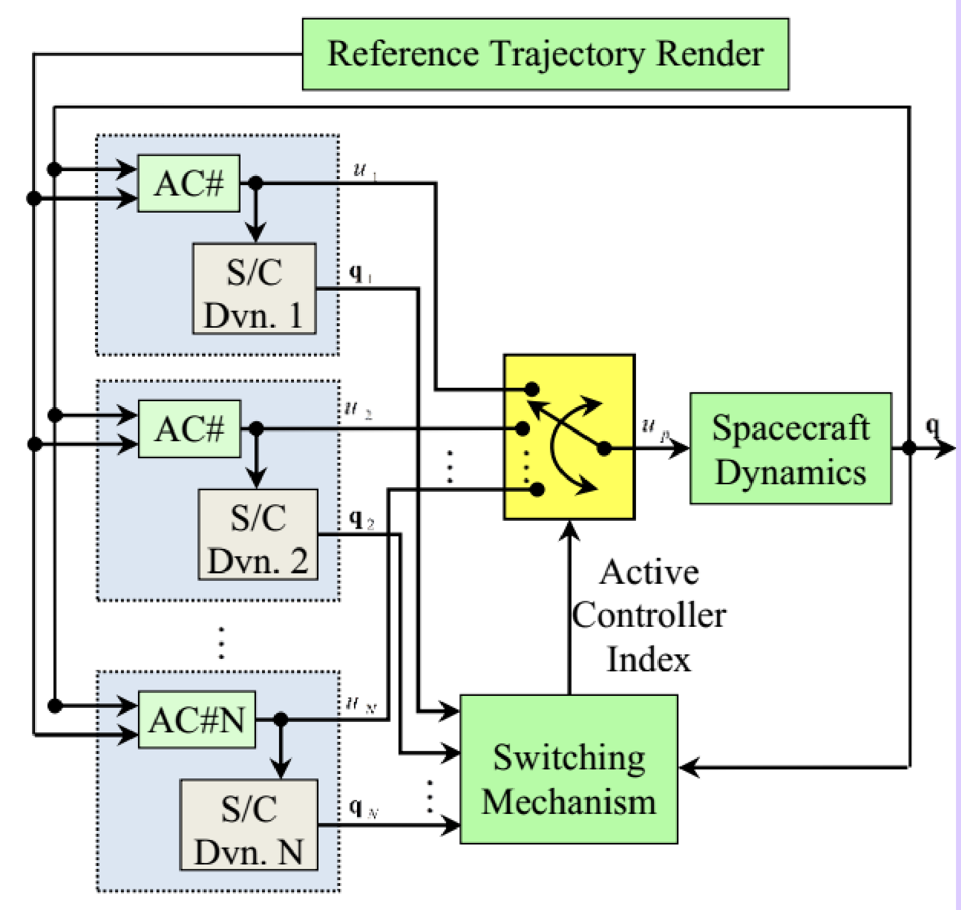
The Proposed Control Scheme

Multiple Model Approach:

Let the parameter space of inertia matrix $\mathbf{I} \in \mathbb{R}^{3\times 3}$ be

$$\mathbf{S} = \{\boldsymbol{\theta} \in \mathbb{R}^6 | \mathbf{I} > 0, l_i \le \theta_i \le u_i, i = 1,2,\ldots,6\}$$

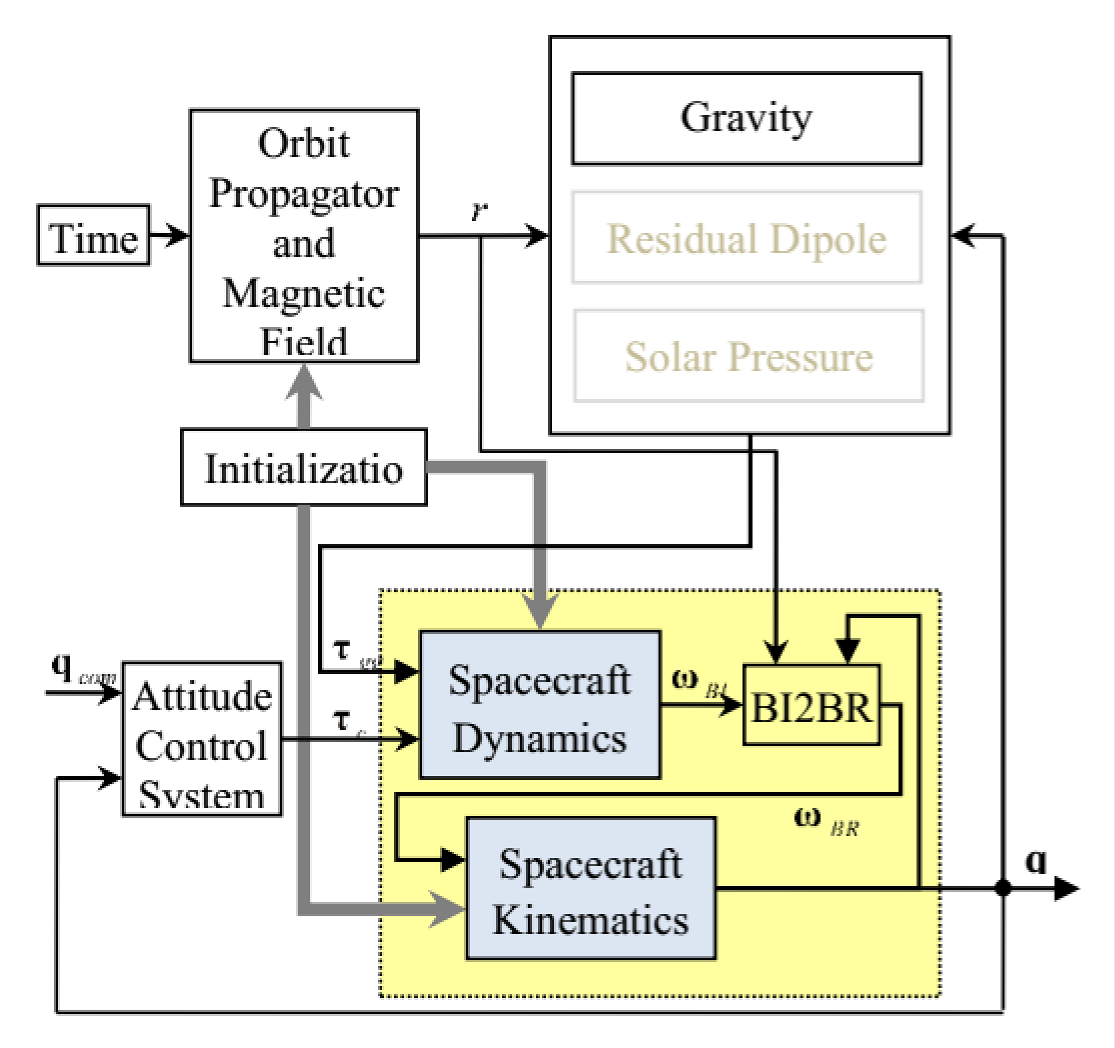
$$\text{active controller index} = \underset{i}{\operatorname{argmin}} \left(\|\mathbf{q}_p - \mathbf{q}_i\|_2 \right)$$



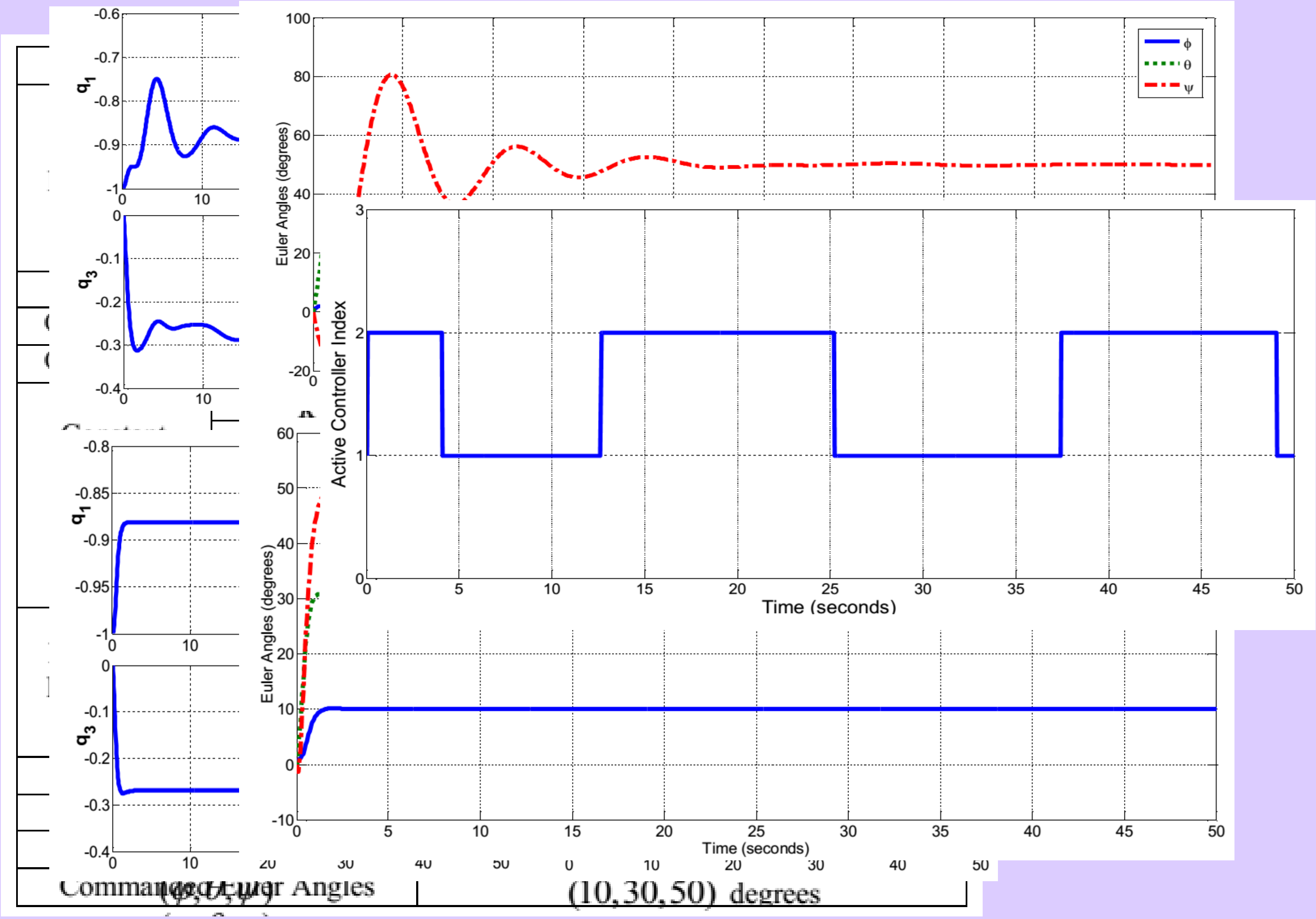
Simulation Results

LEO Satellite Attitude and Orbit Simulator

- Spacecraft Kinematics and Dynamics
- Orbit Propagator
- Earth Magnetic Field
- Space Environment Disturbances
- Attitude Control System



Simulation Results



Conclusion

- an passivity-based output feedback control law is proposed for satellite attitude control under angular velocity constraints
- almost global asymptotic stability of the proposed control law is proved using Lyapunov second method
- The transient response of this control law in the regulation problem was improved significantly by applying the multiple model and switching approach to adaptive control
- A LEO satellite attitude and orbit simulator was developed and used for evaluation of the proposed control scheme

Thank you for your attention